

# Cosmic Ray Momentum Diffusion in Magnetosonic versus Alfvénic Turbulent Fields

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**Abstract.** Energetic particle transport in a finite amplitude magnetosonic and Alfvénic turbulence is considered using Monte Carlo particle simulations, which involve an integration of particle equations of motion. We show that in a low- $\beta$  plasma cosmic ray acceleration can be the most important damping process for magnetosonic waves. Assuming such conditions we derive the momentum diffusion coefficient  $D_p$  for relativistic particles in the presence of anisotropic finite-amplitude turbulent wave fields, for flat and Kolmogorov-type turbulence spectra. We confirm the possibility of larger values of  $D_p$  occurring due to transit-time damping resonance interaction in the presence of isotropic fast-mode waves in comparison to the Alfvén waves of the same amplitude (cf. Schlickeiser & Miller 1998). The importance of quasi-perpendicular fast-mode waves is stressed for the acceleration of high velocity particles.

**Key words:** cosmic rays – magnetohydrodynamic turbulence – interstellar medium – Fermi acceleration

## 1. Introduction

A number of astronomical objects (extragalactic radio sources, supernova remnants, solar flares) emit radiation with non-thermal spectra. These emissions are often connected with the existence of a hot turbulent magnetized plasma providing conditions for particle acceleration by MHD turbulence. It was first shown by Hall & Sturrock (1967) and by Kulsrud & Ferrari (1971) that charged particles can be accelerated by MHD turbulence having wavelengths long compared to the particle gyration radius. For example, the importance of a Fermi-like acceleration mechanism was considered for extragalactic radio sources (Burn 1975; De Young 1976; Blandford & Rees 1978; Achterberg 1979; Eilek 1979). Stochastic acceleration by plasma turbulence is often suggested as the acceleration mechanism for energetic particles during impulsive solar flares (see, e.g. Miller & Ramaty 1987; Steinäcker & Miller 1992; Hamilton & Petrosian 1992). It has been proposed that weak ( $\delta B/B_o \ll 1$ ) MHD waves can simultaneously energize ions by cas-

cading shear Alfvén waves (Miller & Roberts 1995; Miller & Reames 1996) as well as electrons by the accompanying cascading fast-mode waves through a process known as a transit-time acceleration (Miller et al. 1996). The MHD waves are attractive candidates because they can be produced by either large-scale restructuring of the magnetic field, which presumably occurs during the flare energy release phase, or by a shear in the plasma bulk velocity (Roberts et al. 1992), which is likely to be found in regions of reconnection-driven plasma outflows (see, e.g., LaRosa & Moore 1993, Forbes 1996).

The interaction between the waves and particles is determined primarily by delta functions,  $\delta(\omega - k_{\parallel}v_{\parallel} - n\Omega)$ , which select those waves from a spectrum which are resonant with a given particle<sup>1</sup>. At such interactions energies vary in a diffusive way, and over a long timescale a net energy gain results in the process of stochastic acceleration. For  $n \neq 0$ , a particle resonates with those waves that are seen at an integral multiple of a gyrofrequency and for  $n = 0$  at zero frequency. Then, a particle feels a net force which is not averaged out by the phase mixing (Stix 1962). The case of  $n = 0$  provides magnetic analogy to Landau damping and is associated with a first-order change in  $\delta|\vec{B}|$ . This type of coupling is not observed in the case of Alfvén waves but becomes important for the magnetosonic waves containing compressive components of  $\delta\vec{B}$ . The interaction between the particle magnetic momentum and the parallel gradient of the magnetic field is called transit-time damping (cf. discussions by Lee & Völk 1975, Achterberg 1981, Miller et al. 1996).

Recently, Schlickeiser & Miller (1998) presented a quasi-linear derivation of cosmic ray transport coefficients in the presence of MHD waves, including isotropic fast-mode turbulence. For the isotropic Kolmogorov turbulence they demonstrated that the Fokker-Planck coefficients depend both on the transit-time damping and the gyro-resonance interactions. For cosmic ray particles with  $v \gg V_A$  and for vanishing turbulence cross helicity the momentum diffusion coefficient in the fast-mode turbulence is mainly determined by the transit-time damping contribution, leading to a more efficient stochastic acceleration in comparison to the process in the presence of pure Alfvénic turbulence.

The aim of the present paper is to study the momentum diffusion coefficient  $D_p$  in the presence of finite amplitude ( $\equiv \delta B \sim 1$ , here we denote  $\delta B/B_0 \equiv \delta B$ ) magnetosonic and Alfvén waves. The influence of wave anisotropy and of the wave spectrum slope on  $D_p$  is considered. In the next section we discuss the conditions essential for fast-mode wave damping in the plasma. We show that the low- $\beta$  plasma can pro-

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<sup>1</sup> The notation is explained in Appendix A

vide conditions with the main damping process being the cosmic ray particles' acceleration. Then, in Section 3, we summarize the results of quasi-linear analytic derivations of the momentum diffusion coefficient to provide a reference for our numerical modelling. The performed Monte Carlo simulations involving derivations of particles trajectories in the space filled with finite amplitude fast-mode and Alfvén waves are described in Section 4. Anisotropic wave distributions are modeled by choosing their wave vectors from cones directed along the mean magnetic field  $\vec{B}_o$ . For our simulations we adopt fast-mode or Alfvén mode turbulence with the flat- ( $q = 1$ ) or the Kolmogorov ( $q = 5/3$ ) spectrum within the finite wave vector range  $(k_{min}, k_{max})$ . The results are presented and discussed in Section 5. We confirm a substantial increase of  $D_p$  for the fast-mode waves in comparison to the Alfvén waves of the same amplitude if the nearly perpendicular ( $\vec{k} \perp \vec{B}_o$ ) waves are present.

## 2. Dissipation of fast-mode waves in a low $\beta$ plasma

Fast mode waves can effectively accelerate cosmic ray particles, if other dissipation processes are negligible small. We will demonstrate here that in gases with low plasma-beta,  $\beta = 2c_s^2/V_A^2 \ll 1$ , this requirement is fulfilled.

The equilibrium intensity of plasma waves results from the competition of wave generation and wave damping processes. The kinetic equation for a 3-dimensional spectral density  $\bar{W}_i(\vec{k})$ , which denotes the wave energy density per unit volume of wavenumber space of the wave mode  $i$ , is given by the usual conservation equation

$$\frac{\partial \bar{W}_i(\vec{k})}{\partial t} = -\frac{\partial}{\partial \vec{k}} \cdot \vec{F}(\vec{k}) \quad , \quad (2.1)$$

where the flux

$$\vec{F}(\vec{k}) = -D \frac{\partial \bar{W}_i(\vec{k})}{\partial \vec{k}} \quad (2.2)$$

is expressed as a diffusive term with the diffusion coefficient in wavenumber space

$$D = k^2 / \tau_s(k) \quad (2.3)$$

and the spectral energy transfer time scale  $\tau_s(k)$ . This approach, to describe the evolution of turbulence by a diffusion of energy in wavenumber space, was pioneered by Leith (1967) in hydrodynamics, and subsequently introduced to magnetohydrodynamics by Zhou &

Matthaeus (1990). It provides a simple framework to take into account - at least approximately - turbulence evolution in space physics applications. Zhou & Matthaeus (1990) present a general transport equation for the wave spectral density in the case of isotropic turbulence, which includes terms for spatial convection and propagation, nonlinear transfer of energy across the wavenumber spectrum, and a source and sink of wave energy.

For isotropic turbulence one obtains for the associated one-dimensional spectral density  $W_i(k) = 4\pi k^2 \bar{W}_i(\vec{k})$  the simplified diffusion equation

$$\frac{\partial W_i}{\partial t} = \frac{\partial}{\partial k} \left[ \frac{k^4}{\tau_s(k)} \frac{\partial}{\partial k} (k^{-2} W_i) \right] - \Gamma_i W_i + S_i(k) \quad , \quad (2.4)$$

where we included terms for the damping or growth of waves ( $\Gamma_i$ ) and a wave energy injection and/or sink term  $S_i(k)$ .

The spectral energy transfer time scale (or the wavenumber diffusion coefficient (2.3)) depends upon the cascade phenomenology. In the Kolmogorov treatment the spectral energy transfer time at a particular wavelength  $\lambda$  is the eddy turnover time  $\lambda/\delta v$ , where  $\delta v$  is the velocity fluctuation due to the wave. In the so-called Kraichnan treatment the transfer time is longer by a factor  $V_A/\delta v$ . Both phenomenologies are further discussed in Zhou & Matthaeus (1990) and yield

$$\tau_s(k) \simeq \frac{1}{V_A k^{3/2}} \begin{cases} \sqrt{\frac{2U_B}{W_i}} & (\text{Kolmogorov}) \\ \frac{2U_B}{k^{1/2} W_i} & (\text{Kraichnan}) \end{cases} \quad , \quad (2.5)$$

where  $U_B = B_0^2/8\pi$  denotes the energy density of the ordered magnetic field. Substituting these transfer time scales into Eq. (2.4), and assuming a steady state with no damping, we obtain  $W_i = W_0 k^{-q}$ , where  $q = 5/3$  for the Kolmogorov case and  $q = 3/2$  for the Kraichnan phenomenology. The diffusion equation (2.4) in either case is nonlinear.

Besides the generation of turbulence by kinetic cosmic ray streaming instabilities (e.g. Tadamaru 1969), wave cascading from low to high wavenumbers is very often an important way of producing broadband wave spectra. One possibility, discussed e.g. in the context of solar flares (Miller & Roberts 1995), is that long-wavelength turbulence results from the rearrangement of large-scale magnetic fields and/or a shear flow instability, so that it is reasonable to assume the deposition of wave energy peaked at long wavelength, probably comparable to the physical size of the system, as the primary energy release. Cascading as described by the nonlinear diffusion term in Eq. (2.4) will then transfer

this spectral energy to higher wavenumbers, where the waves will be able to resonate with progressively lower energy cosmic ray particles, until they eventually interact with the charged particles in the tail of the background thermal distribution.

According to Ginzburg (1970) the damping rate of fast magnetosonic waves propagating at an angle  $\theta$  with respect to the ordered magnetic field in a thermal electron-proton background plasma is given by

$$\Gamma_t = \sqrt{\frac{\pi\beta}{16}} V_A |k| \frac{\sin^2 \theta}{\cos \theta} F(\beta, \theta) \quad , \quad (2.6)$$

where

$$F(\beta, \theta) = \sqrt{\frac{m_e}{m_p}} + 5 \exp[-(\beta \cos^2 \theta)^{-1}] \quad (2.7)$$

depends on the plasma beta of the background plasma

$$\beta = \frac{2c_S^2}{V_A^2} = \frac{8\pi n_e m_p k_B T_e}{B_0^2} \quad . \quad (2.8)$$

The function  $F(\beta, \theta)$  exhibits an almost perfect "flip-flop" behaviour with constant values  $F(\beta \leq \beta_c) \simeq \sqrt{m_e/m_p} \simeq 1/43$  and  $F(\beta > \beta_c) \simeq 5$  below and above  $\beta_c = 0.5 \cos^{-2} \theta \geq 0.5$ .

The other relevant wave damping process is the transit-time damping acceleration of isotropically distributed cosmic rays. According to Achterberg (1981) the corresponding damping rate is given by

$$\Gamma_r = \frac{\pi V_A}{4c} V_A |k| \frac{\sin^2 \theta}{\cos \theta} \left[ 1 - \frac{V_A^2}{c^2 \cos^2 \theta} \right]^2 \frac{U_p}{U_B} \quad , \quad (2.9)$$

where  $U_B$  and  $U_p$  denote the energy densities of the background magnetic field and the cosmic ray particles, respectively.

For small wave intensities we can neglect the cascading of waves, so that according to Eq. (2.4) the equilibrium fast-mode intensity is simply given by

$$W_f(k) = \frac{S_f(k)}{\Gamma_t + \Gamma_r} \quad . \quad (2.10)$$

Calculating following Achterberg (1981) the ratio of the two damping rates,

$$R \equiv \frac{\Gamma_t}{\Gamma_r} = \frac{U_B}{U_p} \left[ 1 - \frac{V_A^2}{c^2 \cos^2 \theta} \right]^{-2} \frac{c}{\pi^{1/2} V_A} \beta^{1/2} F(\beta, \theta) \quad , \quad (2.11)$$

we find that this ratio is independent of wavenumber  $k$ , and for small plasma beta  $\beta < 0.5$  almost independent from the propagation angle  $\theta$  since  $V_A \ll c$ . The ratio is well approximated by

$$R(\beta < 0.5) \simeq \frac{U_B}{U_p} \frac{c}{V_A} \sqrt{\frac{\beta}{1836\pi}} = \frac{U_B}{U_p} \frac{c}{c_S} \frac{\beta}{\sqrt{3672\pi}} \quad . \quad (2.12)$$

Values of  $R < 1$  indicate that the fast-mode wave energy is damped by accelerating cosmic ray particles. The condition  $R < 1$  translates into a condition for the plasma beta

$$\beta \leq 0.01 \sqrt{T_5} \frac{U_p}{U_B} \quad , \quad (2.13)$$

where  $T_5 = (T/10^5 \text{K})$  denotes the plasma temperature in units of  $10^5 \text{K}$ . In the case of equipartition between magnetic field and cosmic rays,  $U_p/U_B \simeq 1$  we find values of  $R$  less unity if  $\beta \leq 0.01 \sqrt{T_5}$ . In conclusion, in a low  $\beta$ -plasma the fast-mode waves are predominantly dissipated by accelerating cosmic rays by transit-time damping. The inclusion of wave cascading does not qualitatively modify this conclusion as the numerical solutions of Miller et al. (1996) indicate.

### 3. Quasi-linear momentum diffusion coefficient

The quasi-linear theory treats the effect of the weakly perturbed magnetic field as perturbations of orbits of particles moving in the average background field. Schlickeiser (1989) considers quasilinear transport and acceleration parameters for cosmic ray particles interacting resonantly with the Alfvén waves propagating along the average magnetic field. The transport equation can be derived from the Fokker-Planck equation by a well-known approximate scheme (Jokipii 1966, Hasselmann & Wibberenz 1968) which is commonly referred to as the diffusion-convection equation for the pitch-angle averaged phase space density. The electromagnetic fields generated by MHD waves enter into the equation through the Lorentz force term. For fast ( $v \gg V_A$ ) cosmic ray particles, a vanishing cross helicity state of the Alfvén waves and the power-law turbulence spectrum with  $q \geq 1$ ,

$$\bar{W}_i(\vec{k}) = \bar{W}_o^i (\delta B_i)^2 k^{-q} \quad \text{for} \quad (k_{min} < k < k_{max}) \quad (3.1)$$

the momentum diffusion coefficient

$$D_p = \frac{\pi \bar{W}_o}{q(q+2)} \left( \frac{\delta B}{B} \right)^2 |\Omega|^{2-q} \frac{V_A^2 p^2}{v^{3-q}} \quad , \quad (3.2)$$

where

$$\bar{W}_o = \frac{1-q}{k_{max}^{1-q} - k_{min}^{1-q}} \quad \text{for } q > 1$$

and

$$\bar{W}_o = \left( \ln \frac{k_{max}}{k_{min}} \right)^{-1} \quad \text{for } q = 1 \quad .$$

Recently, Schlickeiser & Miller (1998) considered cosmic ray particles interacting with oblique fast-mode waves propagating in a low- $\beta$  plasma. In the cold plasma limit the fast and slow magnetosonic waves degenerate to the fast-mode waves with the dispersion relation  $\omega^2 = V_A^2 k^2$ . The momentum diffusion coefficient (in the original paper ‘ $a_2$ ’ for our  $D_p$ ) and the spatial diffusion coefficient  $\kappa_{\parallel}$  can be calculated as respective pitch angle averages of the Fokker-Planck coefficient  $D_{\mu\mu}(\mu, p)$ . Adopting isotropic fast-mode turbulence with a power-law turbulence spectrum (3.1) they obtained

$$D_{\mu\mu} = \frac{\pi |\Omega| \bar{W}_o (1 - \mu^2)}{4} \left( \frac{\delta B}{B} \right)^2 (R_L)^{q-1} [f_T(\mu) + f_G(\mu)] \quad , \quad (3.3)$$

where  $D_{\mu\mu}$  includes a sum of the transit-time damping ( $f_T$ ) and the gyroresonance ( $f_G$ ) interaction contributions. A considered form of  $f_T$  admits the pitch angle scattering by transit-time damping of super-Alfvénic particles with pitch-angles contained in the range  $\epsilon \leq |\mu| \leq 1$ , where  $\epsilon \equiv V_A/v$ . In the interval  $|\mu| \leq \epsilon$ , where no transit-time damping occurs, the gyroresonance interactions provide a small but finite contribution to the particle scattering rate. As a result the momentum diffusion coefficient  $D_p$  is mainly determined by the transit-time damping interactions and the spatial diffusion coefficient by the gyroresonance interactions. For  $1 \leq q \leq 2$  fast-mode turbulence spectra (Eq. 3.1) they obtained

$$D_p \simeq \frac{\pi \bar{W}_o C_1}{4} \left( \frac{\delta B}{B} \right)^2 |\Omega| (R_L)^{q-1} \frac{V_A^2 p^2}{v^2} \ln \frac{v}{V_A} \quad , \quad (3.4)$$

where

$$C_1 = 2^{1-q} \frac{q \Gamma(q) \Gamma(2 - \frac{q}{2})}{(4 - q^2) \Gamma^3(1 + \frac{q}{2})} \quad . \quad (3.5)$$

#### 4. Description of simulations

The approach applied in the present paper for modelling the particle momentum diffusion is based on numerical Monte Carlo simulations. The general procedure is quite simple: test particles are injected at random positions into a turbulent magnetized plasma and their trajectories are followed by integration of particle equations of motion. Due to the presence of waves, particles move diffusively in configuration and momentum space. By averaging over a large number of trajectories one derives the diffusion coefficients for turbulent wave fields. In the simulations we consider relativistic particles with  $v \gg V_A$  and use dimensionless units (cf. Appendix A):  $\delta B \equiv \delta B/B_o$  for magnetic field perturbations,  $1/\Omega_o$  for time,  $k/k_{res}$  for wave vectors and  $p_o^2\Omega_o$  for the momentum diffusion coefficient. Below, in all our simulations we adopt  $V_A = 10^{-3}c$  and we consider mono-energetic particles with velocity  $v = 0.99c$ . This particular choice allows us to compare the effects of different types of turbulence on the particle momentum diffusion coefficient and, due to the large  $v$ , to evaluate the role of wave anisotropy in the transit-time damping interactions.

##### 4.1. THE WAVE FIELD MODELS

In the modelling we consider a superposition of 384 MHD waves propagating oblique to the average magnetic field  $\vec{B}_o \equiv B_o \hat{e}_z$ . The wave propagation angle with respect to  $\vec{B}_o$  is randomly chosen from a uniform distribution within a cone ('wave-cone') along the mean field. For a given simulation two symmetric cones are considered centered along  $\vec{B}_o$ , with the opening angle  $2\alpha$ , directed parallel and anti-parallel to the mean field direction. The same number of waves is selected from each cone in order to model the symmetric wave field. Related to the  $i$ -th wave, the magnetic field fluctuation vector  $\delta\vec{B}^{(i)}$  is given in the form:

$$\delta\vec{B}^{(i)} = \delta\vec{B}_o^{(i)} \sin(\vec{k}^{(i)} \cdot \vec{r} - \omega^{(i)}t) \quad , \quad (4.1)$$

where  $\delta\vec{B}_o^{(i)}$  is a constant vector selected for a given wave. The electric field fluctuation related to a particular wave is given as  $\delta\vec{E}^{(i)} = -\vec{V}^{(i)} \times \delta\vec{B}^{(i)}$ . For Alfvén waves ('A') we use the dispersion relation

$$\omega_A^2 = k_{\parallel}^2 V_A^2 \quad , \quad (4.2)$$

where  $V_A = B_o/\sqrt{4\pi\rho}$  is the Alfvén velocity in the field  $B_o$ . The wave magnetic field polarization is defined by the formula

$$\delta\vec{B}_A = \delta B_A(\vec{k}, \omega_A) (\vec{k} \times \hat{e}_z) k_{\perp}^{-1} \quad . \quad (4.3)$$



In a low- $\beta$  plasma the fast-mode magnetosonic waves ('M') propagate with the Alfvén velocity and the respective relations are:

$$\omega_M^2 = k^2 V_A^2 \quad , \quad (4.4)$$

$$\delta \vec{B}_M = \delta B_M(\vec{k}, \omega_M) (\vec{k} \times (\vec{k} \times \hat{e}_z)) k^{-1} k_{\perp}^{-1} \quad . \quad (4.5)$$

One should be aware of the fact that the considered turbulence model is unrealistic at large  $\delta B$  and the present results can not be considered as the exact quantitative ones. In particular, in the presence of a finite amplitude turbulence the magnetic field pressure is larger than the mean field pressure and the wave phase velocities can be greater than  $V_A(B_o)$  assumed here.

#### 4.2. SPECTRUM OF THE TURBULENCE

In our simulations we consider the power-law turbulence spectrum in the wave vector range  $(k_{min}, k_{max})$ . The amplitude of the irregular component of the magnetic field, obtained from the energy density defined in equation (3.1), can be written as

$$\delta B(k) = \delta B(k_{min}) \left( \frac{k}{k_{min}} \right)^{-q/2} \quad , \quad (4.6)$$

where  $k_{min} = 0.08$  ( $k_{max} = 8.0$ ) corresponds to the considered longest (shortest) wavelength, respectively, and  $q$  is the wave spectral index. In the present simulations we consider the flat spectrum with  $q = 1$  and the Kolmogorov spectrum with  $q = 5/3$ . We included the flat spectrum because of our earlier simulations (Michalek & Ostrowski 1996), where the Alfvén waves with  $q = 1$  were considered. It provides also a convenient limiting reference for comparison of results for different wave spectra flatter than the Kolmogorov one (e.g. the Kraichnan spectrum or the perturbations' spectra at wavelengths longer than the ones characteristic for the inertial range of the MHD cascade). On the other hand such a turbulence spectrum is very convenient for numerical simulations due to presence of a substantial power in short waves. For the flat spectrum the wave vectors are drawn in a random way from the respective ranges:  $2.0 \leq k \leq 8.0$  for 'short' waves,  $0.4 \leq k \leq 2.0$  for 'medium' waves and  $0.08 \leq k \leq 0.4$  for 'long' waves. The respective wave amplitudes  $\delta B_o$  are drawn in a random manner so as to keep constant

$$\sum_{i=1}^{384} (\delta B_o^{(i)})^2 \equiv \delta B^2, \quad (4.7)$$

where  $\delta B$  is a model parameter, and, separately in all mentioned wave-vector ranges

$$\sum_{i=1}^{128} (\delta B_o^{(i)})^2 \equiv \sum_{i=129}^{256} (\delta B_o^{(i)})^2 \equiv \sum_{i=257}^{384} (\delta B_o^{(i)})^2 \equiv \frac{\delta B^2}{3} \quad . \quad (4.8)$$

Thus the wave energy is uniformly distributed over the considered wave-vector ranges. As a second, more realistic turbulence model we consider one involving the Kolmogorov spectrum. The observed spectra in interplanetary space often have such a form (e.g. Jokipii 1971) and this case is most often discussed in the literature. Here all 384 wave vectors are drawn in a random manner from the whole considered range ( $0.08 \leq k \leq 8.0$ ), but the amplitudes  $\delta B^{(i)}$  are fitted according to the Kolmogorov distribution (Eq. 4.6 with  $q = 5/3$ ) and scaled to keep the formula (4.7) valid. In such turbulence most of energy is carried by long waves.

In the discussion below we will consider four different turbulence fields labeled as follows:

- i. Alfvén waves with the flat spectrum - AF,
- ii. Alfvén waves with the Kolmogorov spectrum - AK,
- iii. Fast-mode waves with the flat spectrum - MF,
- iv. Fast-mode waves with the Kolmogorov spectrum - MK.

## 5. Results

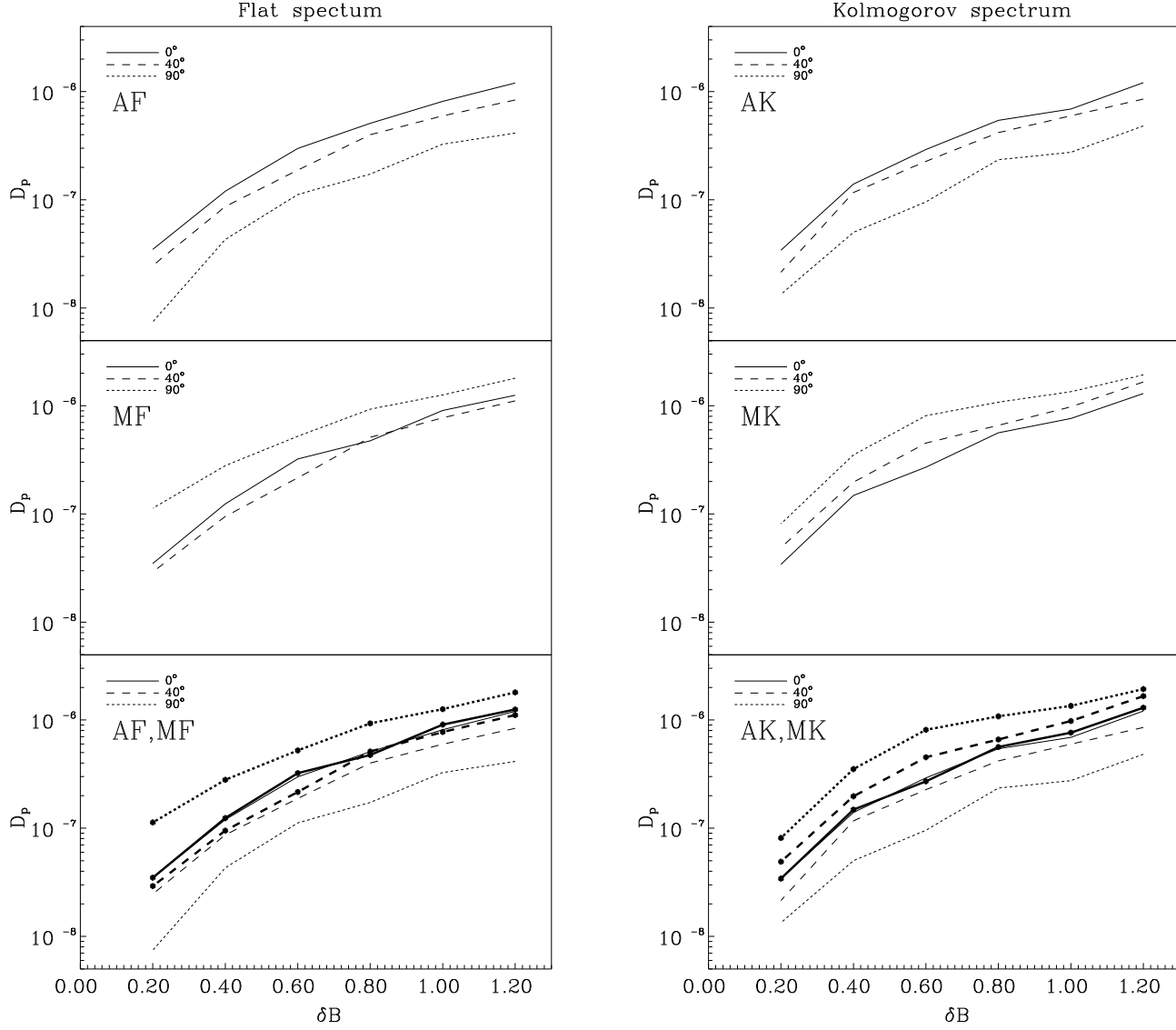
In the present numerical simulations we consider a number of physical situations – turbulence models with finite wave amplitudes and a changing degree of wave anisotropy – not described by analytic means. The results are illustrated in figures 1 and 2. First, the simulated momentum diffusion coefficients,  $D_p$ , for the Alfvén and the magnetosonic turbulence are presented in Fig. 1.: in the upper panels variation of  $D_p$  versus the perturbation amplitude  $\delta B$  for the Alfvén turbulence and in the middle panels for the fast-mode turbulence. To enable comparison these results are superimposed in bottom panels. The derived momentum diffusion coefficients are given for different wave-cone opening angles. The models with  $\alpha = 0^\circ$ ,  $\alpha = 40^\circ$  and  $\alpha = 90^\circ$  represent degenerated one-dimensional, anisotropic and isotropic wave vector distributions, respectively. For clarity, only the results for the mentioned three values are presented, but we also performed simulations for other intermediate wave cone openings:  $\alpha = 30^\circ$  and  $60^\circ$ . The results of these simulations are consistent with the ones described below.

At Fig. 1, in all cases a systematic increase of  $D_p$  with  $\delta B$  can be observed. Usually, it follows the quasi-linear relation  $D_p \propto \delta B^2$ , except for the magnetosonic waves with  $\alpha = 90^\circ$ , where the increase rate seems to be smaller at least at large amplitudes, where it is roughly  $\propto \delta B^{1.5}$ .

For the Alfvén wave turbulence an increase of the wave-cone opening angle  $\alpha$  providing waves with smaller phase velocities leads to decreasing  $D_p$ . The trend is independent of the considered turbulence amplitude and the considered spectrum. In the case of fast-mode waves a more complicated relation is observed. For the flat turbulence spectrum a small increase of the angle  $\alpha$  does not lead to a significant variation of  $D_p$ , and the change is to smaller values of  $D_p$ , like for the Alfvén waves. Only the appearance of waves with wave vectors nearly perpendicular to the mean magnetic field changes this trend leading to an increase of  $D_p$  (by a factor of three for  $\alpha = 90^\circ$  in comparison to  $\alpha = 0^\circ$  at  $\delta B = 0.2$ ). For the Kolmogorov turbulence the wave cone opening-angle increase in the MK model is always followed by an increase of  $D_p$  (in our simulations small values of  $\alpha$ , between  $0^\circ$  and  $30^\circ$ , were not considered) and again it reaches the maximum in the presence of perpendicular waves.

In order to explain this non-monotonic behavior one can refer to the quasi-linear derivations of Schlickeiser & Miller (1998). For  $n = 0$  the resonance condition for the transit-time damping may be written as  $v_{\parallel} = \omega/k_{\parallel}$  and, with the dispersion relation (4.4),  $V_A/v_{\parallel} = k_{\parallel}/k$ . It is clear that for  $V_A \ll v$  particles can efficiently interact with waves at a wide range of  $v_{\parallel}$  only if the waves with  $k_{\perp} \gg k_{\parallel}$  are present. This fact can explain the effective transit-time damping interactions for magnetosonic waves with isotropic distribution. Then, the observed  $D_p$  increase results mainly from the presence of a small fraction of waves propagating quasi-perpendicular to the mean magnetic field. One should note that our simulations were performed for relativistic particles with  $v = 0.99c \gg V_A$ , where the effect is quite pronounced.

In order to verify this behaviour in more detail we considered interaction of such relativistic particles with waves propagating in narrow pitch angle ranges with respect to the mean magnetic field: we performed simulations for wave vector inclinations selected from the angular ranges ( $\pm 5^\circ$ ) around the cones with  $\alpha = 0^\circ$ ,  $45^\circ$  and  $85^\circ$  (Fig. 2). It is clear from the figure that only the presence of waves with wave vectors perpendicular to the mean magnetic field leads to a significant increase of  $D_p$ . For these waves the transit-time damping resonance provides the wave-particle coupling, while the waves propagating nearly parallel to the average magnetic field contribute preferentially due to the gyroresonance. Of course, our reference to the calculations involving various resonances is only approximate, as in the presence of high



*Figure 1.* Variation of the momentum diffusion coefficient  $D_p$ , given in units of  $p_o^2 \Omega_o$ , versus the perturbation amplitude  $\delta B$  and the wave propagation anisotropy. The results are presented for the flat and the Kolmogorov spectra, separately for the Alfvén wave and the fast-mode turbulence. For comparison the results for the Alfvén wave turbulence (thin lines) and the fast-mode turbulence (thick lines with indicated simulation points) are superimposed at the bottom panels.

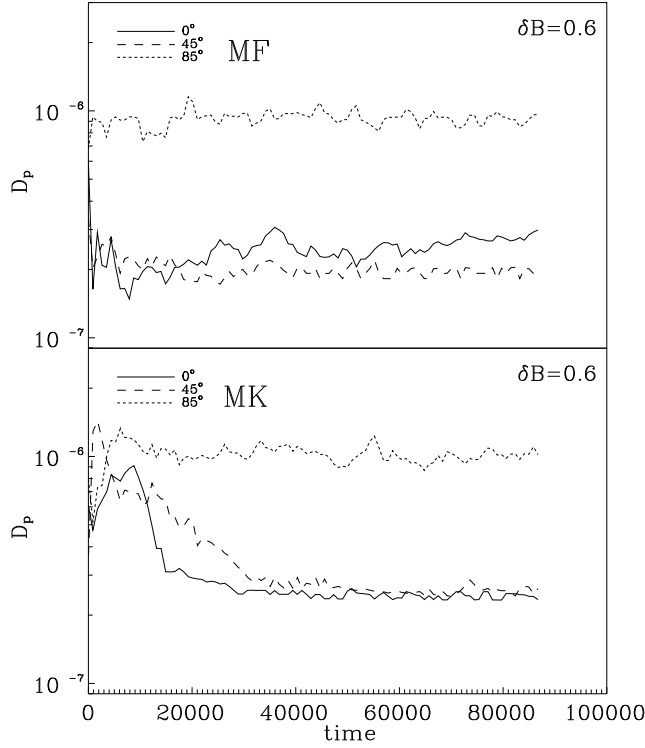


Figure 2. Examples of the simulated  $D_p$ , given in units  $p_o^2 \Omega_o$ , versus the simulation time for waves selected from the respective  $(0^\circ \pm 5^\circ, 45^\circ \pm 5^\circ, 85^\circ \pm 5^\circ)$  pitch angle ranges for both types of turbulence spectrum at  $\delta B = 0.6$ .

amplitude waves the quasi-linear delta form of interaction changes into the ‘broadened resonance’, as discussed by Karimabadi et al. (1992). The simulation errors can be evaluated from comparison at Figs 1,2 of the ‘M’ and ‘A’ results for  $\alpha = 0^\circ$ , which should coincide.

## 6. Final remarks

We considered the momentum diffusion coefficient  $D_p$  in the presence of oblique Alfvén and magnetosonic waves with a wide range of amplitudes, from medium, up to non-linear ones. The influence of the degree of wave anisotropy and the waves’ spectral index was also studied. As expected, in all cases a systematic increase of  $D_p$  with the wave amplitude is observed. We confirm a substantial increase of  $D_p$  for the fast-mode waves in comparison to the Alfvén waves of the same amplitude, if

nearly perpendicular fast-mode waves are present. The effect is caused by the transit-time damping interactions which occur in the presence of magnetosonic waves containing a compressive component of  $\delta\vec{B}$ . For the isotropic fast-mode Kolmogorov turbulence (cf. Eq-s 3.2,4)  $D_p$  is expected to be on a factor of  $\ln(V/V_A)$ , multiplied by a term depending on the index  $q$ , larger than the one for Alfvén waves propagating parallel to  $\vec{B}_o$ . The ratio of these values is about three in our simulations for small  $\delta B$ , which confirms (within the simulation errors) the quasi-linear result. In simulations for the isotropic Kolmogorov spectra  $D_p^{MK}/D_p^{AK} \approx 10$  is roughly constant for a wide range of waves amplitudes. For the isotropic MK and MF models one observes that the transit-time damping scattering weakens with an increasing  $\delta B$ . For these models,  $D_p \propto (\delta B)^{1.5}$ , when some particles can become timely trapped between the compressive waves and the acceleration due to the transit-time damping becomes less efficient.

Fast-mode waves can effectively accelerate cosmic ray particles if other dissipation processes heating the plasma are negligibly small. In Section 2 we demonstrated that for a low beta-plasma,  $\beta \ll 1$ , the fast-mode waves can be predominantly dissipated by accelerating cosmic rays through the transit-time damping mechanism. In addition, the momentum diffusion enhancement due to presence of fast-mode waves could also work effectively in a volume with the turbulence generation force acting. For example, in the vicinity of a strong shock, where the conditions with  $U_p \gg U_B$  can occur (cf. Drury 1983), or in a region of magnetic field reconnection the required fast-mode perpendicular waves are expected to be effectively created. One should remember that the waves' phase velocities are expected to be larger in realistic turbulence than  $V_A(B_o)$  assumed here. Therefore our results for large  $\delta B$  should rather be considered as the lower limits for  $D_p$ .

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# References

- Achterberg A., 1979, A&A, 76, 276
- Achterberg A., 1981, A&A, 97, 259
- Blandford R.D., Rees M.I., 1978, Physica Scripta, 17, 3543
- Burn B.I., 1975, A&A, 45, 435
- De Young D.S., 1976, ARAA, 14, 447
- Drury L.O'C., 1983, Rep. Prog. Phys., 46, 973
- Eilek J.A., 1979, ApJ, 230, 373
- Forbes T.G., 1996, High-Energy Solar Physics, ed. R.Ramaty, N.Mandzhavidze & X.-M.Hua (New York: AIP), 275
- Ginzburg, V. L., 1970, *The propagation of electromagnetic waves in plasmas*, Pergamon Press, Oxford
- Hall D.E., Sturrock P.A., 1967, Phys. Fluids, 10, 2620
- Hamilton R.J., Petrosian V., 1992, ApJ, 398, 350
- Hasselmann K., Wibberenz G., 1967, Z. Geophys., 34, 353
- Jokipii J.R., 1966, ApJ, 146, 480
- Jokipii J.R., 1971, Rev. Geophys. Space Phys., 9, 27
- Karimabadi H., Krauss-Varban D., Terasawa T., 1992, JGR 97, 13853
- Kulsrud R.M., Ferrari A., 1971, ASS, 12, 302
- LaRosa T.N., Moore R.L., 1993, ApJ, 418, 912
- Lee M.A., Völk H.J., 1975, ApJ, 198, 485
- Leith, C. E., 1967, Phys. Fluids, 10, 1409
- Melrose D.B., 1974, Sol. Phys., 37, 353
- Michalek G., Ostrowski M., 1996, Nonlinear Processes in Geophysics 3, 66
- Michalek G., Ostrowski M., 1997, A&A, 326,793
- Miller J.A., Ramaty R., 1987, Sol. Phys., 113, 195
- Miller J.A., Roberts, D.A., 1995, ApJ, 452, 912
- Miller J.A., LaRosa T.N., Moore R.L., 1996, ApJ, 461, 445
- Miller J.A., Reames D.V. 1996, High-Energy Solar Physics, ed. R.Ramaty, N.Mandzhavidze & X.-M.Hua (New York: AIP), 450
- Ramaty R., Kozlovsky B., Lingenfelter R.E., 1979, ApJS, 40, 487
- Roberts D.A., Goldstein M.L., Mattheaus W.H., Ghosh S., 1992 JGR, 97, 17115
- Schlickeiser R., 1989, ApJ, 336, 243
- Schlickeiser R., Miller J.A., 1998, ApJ, 492, 352
- Steinäcker J., Miller J.A., 1992, ApJ, 393, 764
- Stix T.H., 1962, *The Theory of Plasma Waves* McGraw-Hill, New York
- Tademaru E., 1969, ApJ, 158, 958

Zhou, Y., Matthaeus, W. H., 1990, JGR, 95, 14881

## Appendix

### A. Summary of notation

$\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}$  – a magnetic induction vector  
 $\mathbf{B}_0$  – a regular component of the background magnetic field  
 $c$  – the light velocity  
 $c_s$  – a velocity of sound  
 $D_p \equiv a_2$  – a momentum diffusion coefficient  
 $D_{\mu\mu}$  – a pitch-angle diffusion coefficient  
 $e$  – a particle charge  
 $\mathbf{E}$  – an electric field vector  
 $\vec{F}(\vec{k})$  – an energy flux of waves  
 $k$  – a wave vector  
 $k_{res} = 2\pi/r_g$  – a resonance wave vector  
 $k_{\parallel}$  – a wave vector component along  $\vec{B}_0$   
 $m_e$  – the electron mass  
 $m_p$  – the proton mass  
 $m$  – a particle mass  
 $n_e$  – a concentration of electrons  
 $\mathbf{p}$  – a particle momentum vector ( $|\vec{p}| = p$ )  
 $q$  – a wave spectral index  
 $r_g \equiv c/|\Omega|$  – a particle maximum Larmour radius  
 $R_L = v/|\Omega|$  – a particle Larmour radius  
 $R \equiv \Gamma_t/\Gamma_r$   
 $S_i(k)$  – an injection or a sink term for wave energy  
 $U_B \equiv B_0^2/8\pi$  – the energy density of the ordered magnetic field  
 $U_p$  – the energy density of cosmic ray particles  
 $\mathbf{v}$  – a particle velocity vector ( $|\vec{v}| \equiv v$ )  
 $V_A$  – the Alfvén velocity in the field  $B_0$   
 $v_{\parallel}$  – a particle velocity along the mean magnetic field  
 $\bar{W}_i(\vec{k})$  – a 3-dimensional spectral density for the wave mode ‘ $i$ ’  
 $\alpha$  – a wave-cone opening angle  
 $\beta = 2c_s^2/V_A^2$  – a plasma-beta parameter  
 $\delta\mathbf{B}$  – a turbulent component of the magnetic field  
 $\delta v$  – a characteristic velocity fluctuation of the wave  
 $\gamma \equiv (1 - v^2/c^2)^{-1/2}$  – the particle Lorentz factor  
 $\Gamma_i$  – a damping or a growth term for the wave mode ‘ $i$ ’  
 $\Gamma_r$  – a damping rate due to the transit-time acceleration



$\Gamma_t$  – a damping rate in a thermal electron-proton plasma  
 $\omega$  – a wave frequency  
 $\omega_A$  – a frequency of the Alfvén wave  
 $\omega_M$  – a frequency of the magnetosonic wave  
 $\Omega \equiv eB/\gamma mc$  – a particle angular velocity  
 $\Omega_o \equiv eB_o/mc$   
 $\mu \equiv \cos \Theta$   
 $\theta$  – a wave propagation angle with respect to  $\mathbf{B}_o$   
 $\Theta$  – a momentum pitch-angle with respect to  $\mathbf{B}_0$   
 $\tau_s(k)$  – a spectral energy-transfer time scale  
 $\epsilon \equiv V_A/v$

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